



ST. JOSEPH'S COLLEGE, PRAYAGRAJ

HALF YEARLY EXAMINATION 2024

MATHEMATICS

CLASS - X

TIME: 3 Hours

MM: 80

Answers to this Paper must be written on the paper provided separately.

You will not be allowed to write during the first 15 minutes.

This time is to be spent in reading the question paper.

Attempt all questions from Section A and any four questions from Section B

All working, including rough work, must be clearly shown, and must be done on the same sheet as the rest of the answer.

Omission of essential working will result in loss of marks.

The intended marks for questions or parts of questions are given in brackets []

Mathematical tables are provided.

SECTION - A

(Attempt all questions from this Section)

Q 1) Choose the correct answers to the questions from the given options.

[15]

(Do not copy the question, write the correct answers only)

- If $(x-2)$ is a factor of $x^3 - kx - 12$, then the value of k is-
(a) 3 (b) 2 (c) -2 (d) -3
- Shares of company A, paying 12%, Rs.100 shares are at Rs. 80
 - Shares of company B, paying 12%, Rs. 100 shares are at Rs.100
 - Shares of company C, paying 12%, Rs. 100 shares are at Rs.120

Shares of which company are at premium
(a) company A (b) company B (c) company C (d) company A and C
- A letter is chosen at random from all the letters of the English alphabets. The probability that the letter chosen is a vowel, is
(a) $\frac{4}{26}$ (b) $\frac{5}{26}$ (c) $\frac{21}{26}$ (d) $\frac{5}{24}$
- If 3 is a root of the quadratic equation $x^2 - px + 3 = 0$, then p is equal to
(a) 4 (b) 3 (c) 5 (d) 2
- Naveen deposits Rs. 800 every month in a recurring deposit account for 6 months. If he receives Rs. 4884 at the time of maturity, then the interest he earns is
(a) Rs. 84 (b) Rs. 42 (c) Rs. 24 (d) Rs. 284
- The co-ordinates of the vertices of ΔABC are respectively $(-4, -2)$, $(6, 2)$ and $(4, 6)$. The centroid G of ΔABC is
(a) $(2, 2)$ (b) $(2, 3)$ (c) $(3, 3)$ (d) $(0, -1)$
- The point $(3, 0)$ is invariant under reflection in-
(a) The origin (b) X-axis (c) Y-axis (d) Both X and Y axes
- Assertion (A) The price of a share at any time in the market is called market value.
Reason (R) The market value of a share is not fixed it varies from time to time.
(a) A is true, R is false (b) A is false, R is true
(c) Both are true (d) Both are false.
- If x , 12, 8 and 32 are in proportion, then the value of x is
(a) 6 (b) 4 (c) 3 (d) 2
- Given that $\begin{bmatrix} x \\ y \end{bmatrix} \times A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. The order of matrix A is
(a) 2×2 (b) 1×2 (c) 1×3 (d) 2×1
- A straight line passes through the points P $(2, -5)$ and Q $(4, 3)$. The slope of the line PQ is
(a) 1 (b) 4 (c) 7 (d) 3
- The solution set for the given inequation is $3 - 2x \geq x - 12$, $x \in \mathbb{N}$ is
(a) $\{1, 2, 3, 4\}$ (b) $\{1, 2, 3, 4, 5\}$ (c) $\{3, 4, 5, 6\}$ (d) $\{1, 2, 3, 4, 5, 6\}$

13. $\cot^2\theta - \frac{1}{\sin^2\theta}$ is equal to

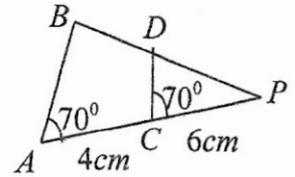
- (a) 1 (b) -1 (c) $\sin^2\theta$ (d) $\sec^2\theta$

14. If the discriminant of $3x^2 + 2x + a = 0$ is double the discriminant of $x^2 - 4x + 2 = 0$, then the value of a is

- (a) 2 (b) -2 (c) 1 (d) -1

15. In the given figure $\angle BAP = \angle DCP = 70^\circ$. $PC = 6\text{cm}$ and $CA = 4\text{cm}$, then $PD : DB$ is

- (a) 5 : 3 (b) 3 : 5 (c) 3 : 2 (d) 2 : 3



Q 2)

- If $(x - 2)$ is a factor of the expression $2x^3 + ax^2 + bx - 14$ and when the expression is divided by $(x - 3)$ it leaves a remainder 52, find the values of a and b . [4]
- A line segment AB meets x-axis at A and y-axis at B. $P(4, -1)$ divides AB in the ratio 1 : 2. Find [4]
 - The coordinates of A and B
 - The equation of the line through P and perpendicular to AB.
- Prove: [4]

$$\frac{\cot\theta + \operatorname{cosec}\theta - 1}{\cot\theta - \operatorname{cosec}\theta + 1} = \frac{1 + \cos\theta}{\sin\theta}$$

Q 3)

- If a, b, c and d are in continued proportion, prove that [4]

$$\sqrt{ab} + \sqrt{bc} - \sqrt{cd} = \sqrt{(a + b - c)(b + c - d)}$$
- Let A be a matrix such that [4]

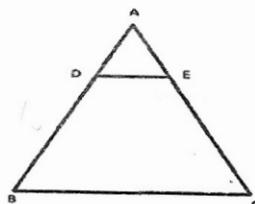
$$\begin{bmatrix} 5 & -2 \\ 1 & 3 \end{bmatrix} \cdot A = \begin{bmatrix} 4 \\ 11 \end{bmatrix}$$
 - Write the order of A
 - Find A
- Use graph sheet to answer this question. Take 2cm = 1 unit along both the axes. [5]
 - Plot A, B, C where A (0, 4), B (1, 1) and C (4, 0).
 - Reflect A and B on the x-axis and name them as E and D respectively.
 - Reflect B through the origin and name it F. Write down the coordinates of F.
 - Reflect B and C on the y-axis and name them as H and G respectively.
 - Join points A, B, C, D, E, F, G, H and A in order and name the closed figure formed.

SECTION-B

(Attempt any four questions from this section)

Q 4)

- Mr. Gupta opened a recurring deposit account in a bank. He deposited Rs.2500 per month for 2 years. At the time of maturity, he got Rs. 67500. Find [3]
 - the total interest earned by Mr. Gupta.
 - the rate of interest per annum.
- Solve the quadratic equation $5x(x+2) = 3$ and give your answer correct to two decimal places. [3]
- In the given figure, $DE \parallel BC$ [4]
 - Prove that $\triangle ADE$ and $\triangle ABC$ are similar
 - Given that $AD = \frac{1}{2}BD$, calculate DE if $BC = 4.5\text{ cm}$



[2]



Q 5)

1. Solve the following inequation and graph the solution set on the number line. [3]

$$-2\frac{2}{3} \leq x + \frac{1}{3} < 3\frac{1}{3}, x \in R.$$

2. Prove the identity $\frac{\sin A}{1+\cos A} = \operatorname{cosec} A - \cot A$ [3]

3. Using componendo and dividendo, find the value of x. [4]

$$\frac{\sqrt{3x+4} + \sqrt{3x-5}}{\sqrt{3x+4} - \sqrt{3x-5}} = 9$$

Q 6)

1. A man invests a sum of money in ₹25 shares, paying 12% dividend and quoted at ₹36. If his annual income from these shares is ₹720, calculate: [3]

- His total investment
- The number of shares bought by him
- The rate of return on his investment

2. Find the ratio in which the line joining A (6, 5) and B (4,-3) is divided by the line y=2. Also, find the coordinates of the point of intersection. [3]

3. From a boat, 200m away from a vertical cliff, the angles of elevation of the top and the foot of a vertical pillar at the edge of the cliff are 36° and 34° respectively. [4]

- Find the height of the cliff
- The height of the pillar

Q 7)

1. A box contain 15 marbles bearing numbers 1, 2, 3, ..., 15 respectively. A marble is drawn at random from the box. Find the probability that the number on the marble is : [3]

- A composite number.
- A number lying between 8 and 12.
- A number divisible by 2 or 5

2. Using factor theorem, factorise completely $6x^3 - 7x^2 - 11x + 12$ [3]

3. Two years ago, a man's age was three times the square of his son's age. In three years' time, his age will be four times his son's age. Find: [4]

- The present age of the man.
- The age of the son two years ago.

Q 8)

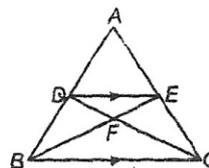
- i. Points A and B have coordinates (7,-3) and (1, 9), respectively. Find [5]

(a) the slope of AB.

(b) the equation of the perpendicular bisector of the line segment AB.

- ii. In the given Figure, ABC is a triangle. DE is parallel to BC and

$$\frac{AD}{BD} = \frac{3}{2}$$



- (a) Determine the ratio $\frac{AD}{AB}$ and $\frac{DE}{BC}$

- (b) Prove that ΔDEF is similar to ΔCBF . Also, find $\frac{EF}{FB}$

[3]



Q 9)

1. Given $A = \begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix}$ and if $A^2 = 9A + mI$, where I is the identity matrix of order 2. Find the value of m . [3]
2. A model of a ship is made to a scale of 1 : 250. Find: [3]
 - a) The length of the ship, if the length of its model is 1.2m
 - b) The area of the deck of the ship, if the area of the deck of its model is $1.6m^2$.
 - c) The volume of its model, when the volume of the ship is 1 cubic kilometre.
3. From the top of a light house 100m high the angles of depression of two ships on opposite sides of it are 45° and 30° respectively. Find the distance between the two ships to the nearest metre. [4]

Q 10)

1. Find the value(s) of k for which the below equation has equal roots. [3]
$$(k + 1)x^2 + 2(k + 3)x + (k + 8) = 0$$
2. Solve the following inequation, write the solution set and represent it on the number line. [3]
$$-2 \leq \frac{1}{2} - \frac{2x}{3} < 1\frac{5}{6}; x \in I$$
3. If $\frac{a^3+3ab^2}{3a^2b+b^3} = \frac{x^3+3xy^2}{3x^2y+y^3}$ using the properties of proportion prove: $\frac{x}{a} = \frac{y}{b}$ [4]